Branching pomsets and event structures (oral communication)

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ICE 2023
Some context

Branching pomsets for choreographies

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Some context

Branching pomsets for choreographies

\[ [a \to b: x ; ((b \to c: x + d \to e: x) || c \to a: x)] \]
Conclusions and future work

Summary

- Branching pomsets
- Compact for both concurrency and choice
- Can express the same behaviour as choreographies

Future work

- Framework improvements: $n$-ary choices, partial order, loops
- Static analysis: realisability

https://arca.di.uminho.pt/b-pomset/
“What about event structures?”
Branching pomsets and event structures
(oral communication)

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ICE 2023
• **Branching pomsets**: a generic model for concurrency
• **Event structures**: a brief overview of the landscape
• **Comparison**: relative expressiveness
Branching pomsets and event structures

**Basis:** partially ordered multisets / pomsets (Pratt 1986)

- a set of events
  above: \( \{a, b, c, d, e, f, g, h\} \)
- a partial order on the events
  above: the reflexive and transitive closure of the arrows
- a labelling function from events to some set of labels
  above: omitted / identity (irrelevant for this talk)
Extension: choices

- expressing choices with pomsets requires a set of pomsets
- with many choices, this set may become exponentially large
- solution: add a representation of choices
**Choice model**: branching structure

- **add branching structure**: a tree whose leaves are the events above: \(\{a, b, g, h, C_1, C_2\}\),
  where \(C_1 = \{\{c\}, \{d\}\}\) and \(C_2 = \{\{e\}, \{f\}\}\)
- **replace the partial order with a precedence relation**, whose reflexive and transitive closure is a partial order above: the arrows
For comparison: the corresponding set of pomsets

\begin{align*}
&\begin{array}{c}
\text{a} \quad \text{c} \quad \text{e} \quad \text{g} \\
\downarrow & \quad \downarrow & \quad \downarrow \\
\text{b} & \quad \text{h} \\
\end{array} & \begin{array}{c}
\text{a} \quad \text{e} \quad \text{g} \\
\downarrow & \quad \downarrow \\
\text{b} & \quad \text{h} \\
\end{array} \\
&\begin{array}{c}
\text{a} \quad \text{c} \quad \text{g} \\
\downarrow & \quad \downarrow \\
\text{b} & \quad \text{f} \quad \text{h} \\
\end{array} & \begin{array}{c}
\text{a} \quad \text{g} \\
\downarrow & \quad \downarrow \\
\text{b} & \quad \text{d} \quad \text{f} \quad \text{h} \\
\end{array}
\end{align*}
Semantics: refining $\Rightarrow$ resolving any number of choices
Branching pomsets

**Semantics:** enabling (followed by firing) $\Rightarrow$ refining s.t. the chosen event is minimal and top-level, resolving no more than necessary.
Branching pomsets

**Semantics:** enabling (followed by firing) $\Rightarrow$ refining s.t. the chosen event is minimal and top-level, resolving no more than necessary.
Also: nested choices
**Choice model**: conflict relation

- add conflict relation; two conflicting events may not occur together in the same execution

above: \{ (c, d), (e, f) \}

- most classes of event structures define variations on causality and/or conflicts
Event structures

**Landscape** (partial): static and dynamic classes of event structures

- Prime → Asymmetric → Growing
- Bundle → Extended Bundle
- Flow → Stable → Dual → Shrinking
- Dynamic Causality
- Resolvable Conflict
- HDES

Arrows represent (strict) inclusion in terms of expressiveness

Figure: Arbach et al., Dynamic causality in event structures (2018)
**Event structures**

**Landscape** (partial): *static* and *dynamic* classes of event structures

Prime ➔ Asymmetric ➔ Growing

Bundle ➔ Extended Bundle

Flow ➔ Stable ➔ Dual ➔ Shrinking

Dynamic Causality ➔ HDES

Resolvable Conflict

Arrows represent (strict) inclusion in terms of expressiveness

Figure: Arbach et al., Dynamic causality in event structures (2018)

**Most relevant for this talk:** growing and shrinking causality ⇒ dynamically adding and removing causalities
Comparison

Prime → Growing

Bundle → Extended Bundle

→ Shrinking

Dynamic Causality

Resolvable Conflict

Branching Pomsets

Tree-like
**Dynamic causality with counters**: replaced dynamic causality event structures with a new variant with nice property; the order of events is irrelevant for the resulting causal state

As a result: uniformly defined semantics for all shown classes
Comparison

**Generic proof**: inclusion in event structures for resolvable conflict of any class of event structures where the causal state is order-independent, including dynamic counters.
Next up: branching pomsets
**Comparison**

Non-inclusion: not all prime event structures expressible as branching pomsets — would need overlapping boxes

```
a . . . c
#  #  #
.  .  .

b   d
```
Comparison

Non-inclusion: not all branching pomsets expressible as growing causality event structures — would need disjunctive causality
Non-inclusion: not all branching pomsets expressible as extended bundle event structures — $c$ can be disabled and then re-enabled.
**Non-inclusion**: not all branching pomsets expressible as shrinking causality event structures — $c$ can be disabled and then re-enabled.
Consequently: branching pomsets incomparable with prime, growing causality, extended bundle and shrinking causality event structures
**Comparison**

**Tree-like** → **Branching Pomsets** → **Prime**

**Bundle** → **Extended Bundle** → **Growing**

**Shrinking** → **Resolvable Conflict** → **Dynamic Counters**

**Inclusion**: subset of branching pomsets, dubbed *tree-like*, can be expressed as prime event structures
Comparison

**Inclusion**: same generic proof as for event structures also holds for branching pomsets; they can all be expressed as event structures for resolvable conflict.
**Inclusion conjecture**: dynamic causality event structures (with counters) may be powerful enough to express all branching pomsets; no proof yet.
Conclusions and future work

Summary

- branching pomsets as a generic model for concurrency
- comparison with various classes of event structures
- interesting behaviour: incomparable with most, included in some more expressive classes of dynamic event structures

Future work

- proving or disproving the dynamic counters conjecture
- study the expressiveness of branching pomsets with overlapping boxes
- expand static analysis of branching pomsets