Branching pomsets for choreographies

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Outline

- Choreographies
- Pomsets
- Branching pomsets
- Choreographies as branching pomsets

Choreographies

$$(\mathsf{a}{\rightarrow}\mathsf{b}{:}\texttt{int}\ ;\ \mathsf{b}{\rightarrow}\mathsf{c}{:}\texttt{bool}) \parallel \mathsf{a}{\rightarrow}\mathsf{c}{:}\texttt{int}$$

Alice communicates an integer to Bob, after which Bob communicates a boolean to Carol. Simultaneously, Alice communicates an integer to Carol.

Choreographies

$$c ::= \mathbf{0} \mid \mathsf{a}{\to}\mathsf{b}{:}\mathtt{x} \mid \boxed{\mathsf{ab}?\mathtt{x}} \mid c \,;\, c \mid c + c \mid c \parallel c \mid c^*$$

Choreographies

$$c ::= \mathbf{0} \mid \mathsf{a}{\rightarrow}\mathsf{b}{:}\mathtt{x} \mid \boxed{\mathsf{ab}?\mathtt{x}} \mid c\,;c \mid c + c \mid c \parallel c \mid c^*$$

Semantics are mostly standard:

$$\begin{aligned}
&(\mathsf{a}{\rightarrow}\mathsf{b}{:}\mathtt{int}\,;\mathsf{b}{\rightarrow}\mathsf{c}{:}\mathtt{bool}) \parallel \mathsf{a}{\rightarrow}\mathsf{c}{:}\mathtt{int}\\
\xrightarrow{\mathsf{ac}!\mathtt{int}}\ &(\mathsf{a}{\rightarrow}\mathsf{b}{:}\mathtt{int}\,;\mathsf{b}{\rightarrow}\mathsf{c}{:}\mathtt{bool}) \parallel \mathsf{ac}?\mathtt{int}\\
\xrightarrow{\mathsf{ab}!\mathtt{int}}\ &(\mathsf{ab}?\mathtt{int}\,;\mathsf{b}{\rightarrow}\mathsf{c}{:}\mathtt{bool}) \parallel \mathsf{ac}?\mathtt{int}\\
\xrightarrow{\mathsf{ab}?\mathtt{int}}\ &\mathsf{b}{\rightarrow}\mathsf{c}{:}\mathtt{bool} \parallel \mathsf{ac}?\mathtt{int}\\
\xrightarrow{\mathsf{bc}!\mathtt{bool}}\ &\mathsf{bc}?\mathtt{bool} \parallel \mathsf{ac}?\mathtt{int}\\
\xrightarrow{\mathsf{ac}?\mathtt{int}}\ &\mathsf{bc}?\mathtt{bool}\\
\xrightarrow{\mathsf{bc}?\mathtt{bool}}\ &\mathbf{0}
\end{aligned}$$

Choreographies

Sequential composition is *weak*:

$$
\begin{array}{rl}
 & \mathtt{a}{\to}\mathtt{b}{:}\mathtt{int}\ ;\ \mathtt{b}{\to}\mathtt{c}{:}\mathtt{bool}\ ;\ \mathtt{a}{\to}\mathtt{c}{:}\mathtt{int} \\
\xrightarrow{\mathtt{ab!int}} & \mathtt{ab?int}\ ;\ \mathtt{b}{\to}\mathtt{c}{:}\mathtt{bool}\ ;\ \mathtt{a}{\to}\mathtt{c}{:}\mathtt{int} \\
\xrightarrow{\mathtt{ac!int}} & \mathtt{ab?int}\ ;\ \mathtt{b}{\to}\mathtt{c}{:}\mathtt{bool}\ ;\ \mathtt{ac?int} \\
\not\xrightarrow{\mathtt{ac?int}} &
\end{array}
$$

Choreographies

Partial termination (Rensink and Wehrheim 2001)

If $c_1 \xrightarrow{\surd_\ell} c_1'$ and $c_2 \xrightarrow{\ell} c_2'$ then $c_1 \,;\, c_2 \xrightarrow{\ell} c_1' \,;\, c_2'$

- If c_1 is independent of the subject of ℓ then $c_1 \xrightarrow{\surd_\ell} c_1$.
- If c_1 can resolve choices to be independent of the subject of ℓ then $c_1 \xrightarrow{\surd_\ell} c_1'$.
- Otherwise $c_1 \not\xrightarrow{\surd_\ell}$.

Choreographies

Partial termination

$$
\begin{aligned}
& \mathsf{a}{\to}\mathsf{b}{:}\mathtt{int} \,;\, \mathsf{b}{\to}\mathsf{c}{:}\mathtt{bool} \,;\, \mathsf{a}{\to}\mathsf{c}{:}\mathtt{int} \\
\xrightarrow{\mathsf{ab}!\mathtt{int}}\ & \mathsf{ab}?\mathtt{int} \,;\, \mathsf{b}{\to}\mathsf{c}{:}\mathtt{bool} \,;\, \mathsf{a}{\to}\mathsf{c}{:}\mathtt{int} \\
\xrightarrow{\mathsf{ac}!\mathtt{int}}\ & \mathsf{ab}?\mathtt{int} \,;\, \mathsf{b}{\to}\mathsf{c}{:}\mathtt{bool} \,;\, \mathsf{ac}?\mathtt{int} \\
\not\xrightarrow{\mathsf{ac}?\mathtt{int}}\ &
\end{aligned}
$$

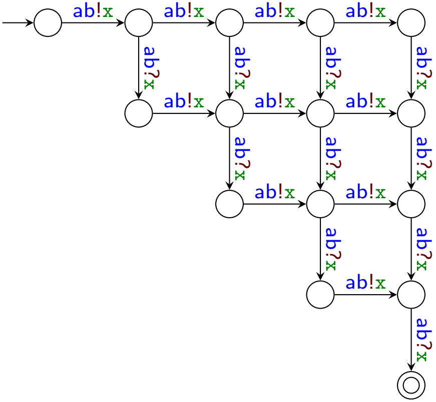
- $\mathsf{ab}?\mathtt{int} \,;\, \mathsf{b}{\to}\mathsf{c}{:}\mathtt{bool} \xrightarrow{\checkmark_{\mathsf{ac}!\mathtt{int}}} \mathsf{ab}?\mathtt{int} \,;\, \mathsf{b}{\to}\mathsf{c}{:}\mathtt{bool}$
- $\mathsf{ab}?\mathtt{int} \,;\, \mathsf{b}{\to}\mathsf{c}{:}\mathtt{bool} \not\xrightarrow{\checkmark_{\mathsf{ac}?\mathtt{int}}}$

Choreographies

Partial termination

- $\mathsf{a}\to\mathsf{b}{:}\mathtt{x} + \mathsf{a}\to\mathsf{c}{:}\mathtt{x} \xrightarrow{\checkmark_{\mathsf{ad}?\mathtt{x}}} \mathsf{a}\to\mathsf{b}{:}\mathtt{x} + \mathsf{a}\to\mathsf{c}{:}\mathtt{x}$
- $\mathsf{a}\to\mathsf{b}{:}\mathtt{x} + \mathsf{a}\to\mathsf{c}{:}\mathtt{x} \xrightarrow{\checkmark_{\mathsf{ba}!\mathtt{x}}} \mathsf{a}\to\mathsf{c}{:}\mathtt{x}$
- $\mathsf{a}\to\mathsf{b}{:}\mathtt{x} + \mathsf{a}\to\mathsf{c}{:}\mathtt{x} \not\xrightarrow{\checkmark_{\mathsf{ba}?\mathtt{x}}}$

a→b:x ; a→b:x ; a→b:x ; a→b:x

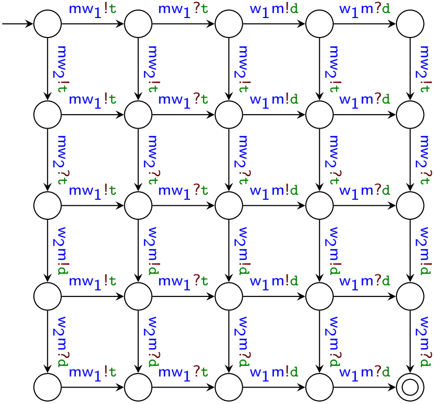


states:

$O(n^2)$

Choreographies

$(m \rightarrow w_1 : t \; ; \; w_1 \rightarrow m : d) \parallel (m \rightarrow w_2 : t \; ; \; w_2 \rightarrow m : d)$



states:

$O(5^n)$

Pomsets

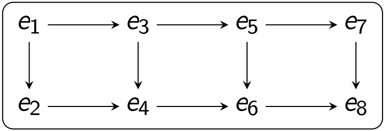
Partially ordered multiset (Pratt 1986)

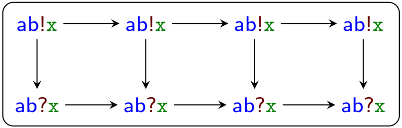
a→b:x ; a→b:x ; a→b:x ; a→b:x

$$\langle \{e_1, \ldots, e_8\}, \qquad O(n)$$

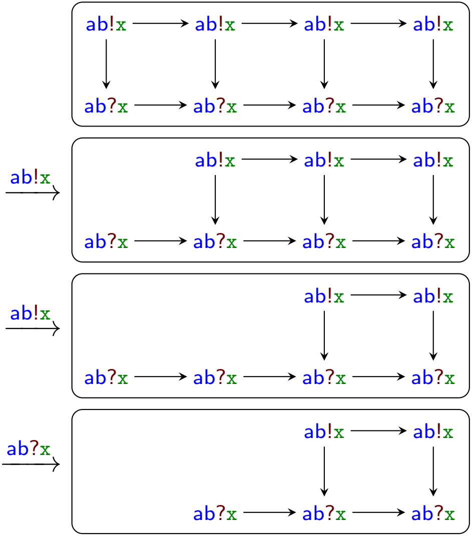
$$\{e_i \le e_j \mid i \le j \wedge (j \text{ is even} \vee i \text{ is odd})\}, \qquad O(n^2)$$

$$e_i \mapsto \begin{cases} \text{ab!x} & \text{if } i \text{ is odd} \\ \text{ab?x} & \text{if } i \text{ is even} \end{cases} \rangle \qquad O(n)$$





Pomsets


ab!x ab!x ab!x ab!x
ab?x ab?x ab?x ab?x
ab!x
ab!x ab!x ab!x
ab?x ab?x ab?x ab?x
ab!x
ab!x ab!x
ab?x ab?x ab?x ab?x
ab?x
ab!x ab!x
ab?x ab?x ab?x

Pomsets

$$(\mathtt{m}{\to}\mathtt{w}_1{:}\mathtt{t} \;;\; \mathtt{w}_1{\to}\mathtt{m}{:}\mathtt{d}) \parallel (\mathtt{m}{\to}\mathtt{w}_2{:}\mathtt{t} \;;\; \mathtt{w}_2{\to}\mathtt{m}{:}\mathtt{d})$$

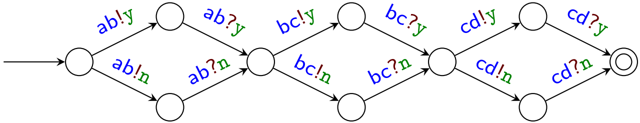
$$\begin{array}{ll}
\langle\{e_1,\dots,e_8\}, & O(n) \\
\;\{e_i \le e_j \mid i \le j \wedge i \equiv j \bmod 2\}, & O(n^2) \\
\;\{e_1 \mapsto \mathtt{mw_1!t}, \dots, e_8 \mapsto \mathtt{w_2m?d}\}\rangle & O(n)
\end{array}$$

$$\begin{array}{l}
e_1 \longrightarrow e_3 \longrightarrow e_5 \longrightarrow e_7 \\
e_2 \longrightarrow e_4 \longrightarrow e_6 \longrightarrow e_8
\end{array}$$

$$\begin{array}{l}
\mathtt{mw_1!t} \longrightarrow \mathtt{mw_1?t} \longrightarrow \mathtt{w_1m!d} \longrightarrow \mathtt{w_1m?d} \\
\mathtt{mw_2!t} \longrightarrow \mathtt{mw_2?t} \longrightarrow \mathtt{w_2m!d} \longrightarrow \mathtt{w_2m?d}
\end{array}$$

Pomsets

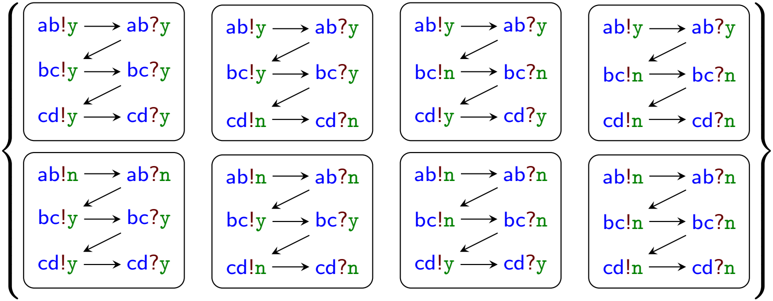
$$(a{\to}b{:}y + a{\to}b{:}n)\ ;\ (b{\to}c{:}y + b{\to}c{:}n)\ ;\ (c{\to}d{:}y + c{\to}d{:}n)$$



$O(n)$ states

Pomsets

(a→b:y + a→b:n) ; (b→c:y + b→c:n) ; (c→d:y + c→d:n)



$O(2^n)$ pomsets

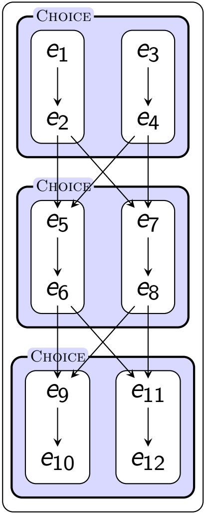
Branching pomsets

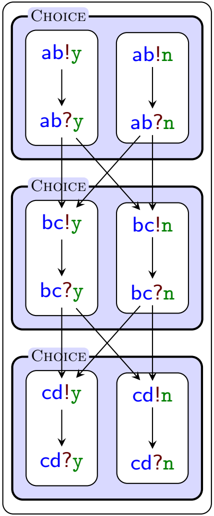
(a→b:y + a→b:n) ; (b→c:y + b→c:n) ; (c→d:y + c→d:n)

Branching pomset: pomset with a branching structure $\mathcal{B}$

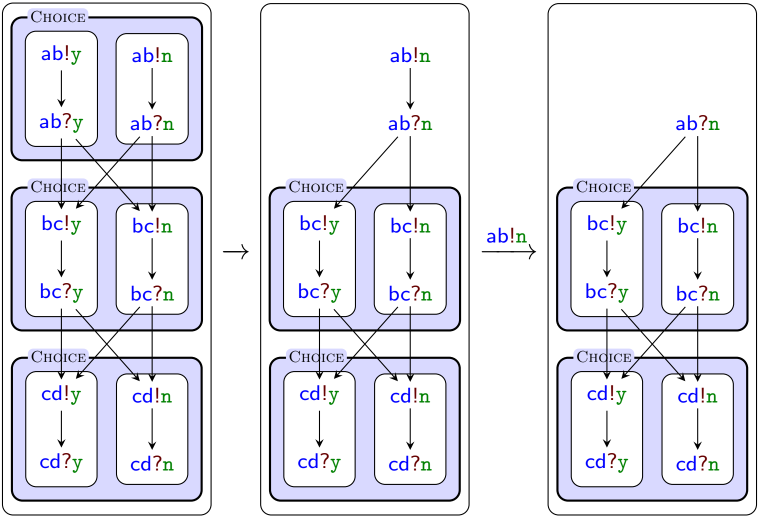
$$\mathcal{B} ::= \{\mathcal{C}_1, \ldots, \mathcal{C}_n\}$$
$$\mathcal{C} ::= e \mid \{\mathcal{B}_1, \mathcal{B}_2\}$$

Here: $\mathcal{B} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$, where
$\mathcal{C}_1 = \{\{e_1, e_2\}, \{e_3, e_4\}\}$,
$\mathcal{C}_2 = \{\{e_5, e_6\}, \{e_7, e_8\}\}$ and
$\mathcal{C}_3 = \{\{e_9, e_{10}\}, \{e_{11}, e_{12}\}\}$.





Branching pomsets



Branching pomsets

$$((\mathsf{a}{\to}\mathsf{b}{:}\mathtt{y} \parallel \mathsf{a}{\to}\mathsf{c}{:}\mathtt{y} \parallel \mathsf{a}{\to}\mathsf{d}{:}\mathtt{y}) + (\mathsf{a}{\to}\mathsf{b}{:}\mathtt{n} \parallel \mathsf{a}{\to}\mathsf{c}{:}\mathtt{n} \parallel \mathsf{a}{\to}\mathsf{d}{:}\mathtt{n}))$$
$$\parallel \; \ldots$$
$$\parallel \; ((\mathsf{d}{\to}\mathsf{a}{:}\mathtt{y} \parallel \mathsf{d}{\to}\mathsf{b}{:}\mathtt{y} \parallel \mathsf{d}{\to}\mathsf{c}{:}\mathtt{y}) + (\mathsf{d}{\to}\mathsf{a}{:}\mathtt{n} \parallel \mathsf{d}{\to}\mathsf{b}{:}\mathtt{n} \parallel \mathsf{d}{\to}\mathsf{c}{:}\mathtt{n}))$$

Finite state machine: huge

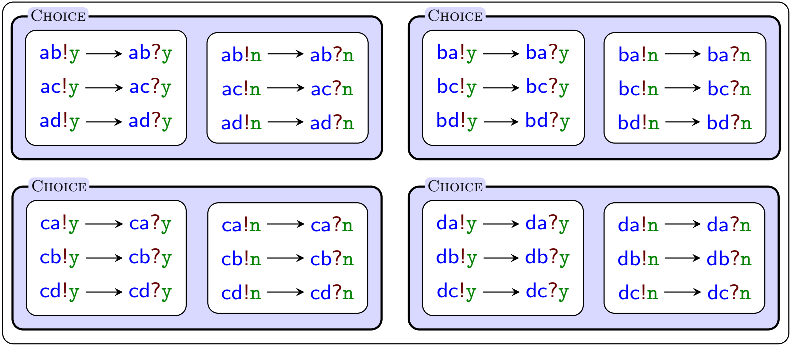
Pomsets: 16 pomsets $\times$ 24 events each $(= 384)$

Branching pomset: . . .

Branching pomsets

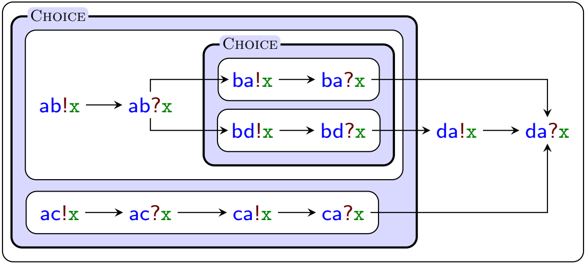
$$((a{\to}b{:}y \parallel a{\to}c{:}y \parallel a{\to}d{:}y) + (a{\to}b{:}n \parallel a{\to}c{:}n \parallel a{\to}d{:}n))$$
$$\parallel \ \ldots$$
$$\parallel ((d{\to}a{:}y \parallel d{\to}b{:}y \parallel d{\to}c{:}y) + (d{\to}a{:}n \parallel d{\to}b{:}n \parallel d{\to}c{:}n))$$

Branching pomset: 48 events



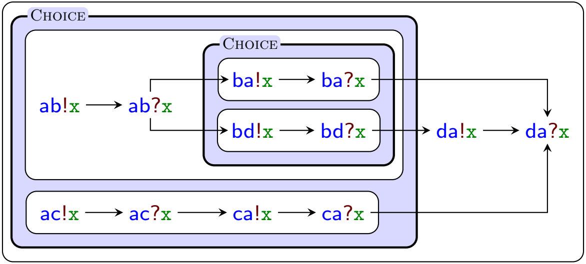
Branching pomsets

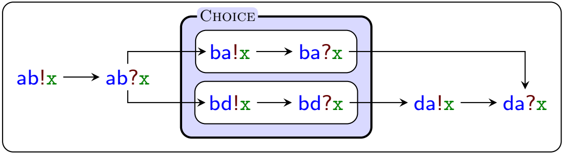
Refining: resolving (any number of) choices



Branching pomsets

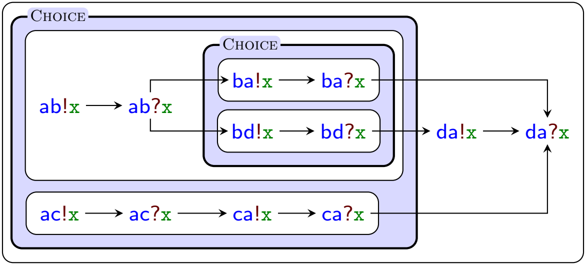
Refining: resolving (any number of) choices

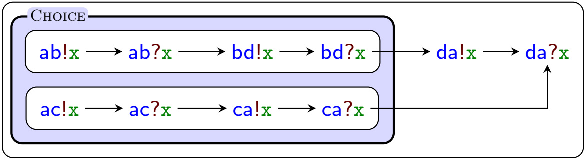




Branching pomsets

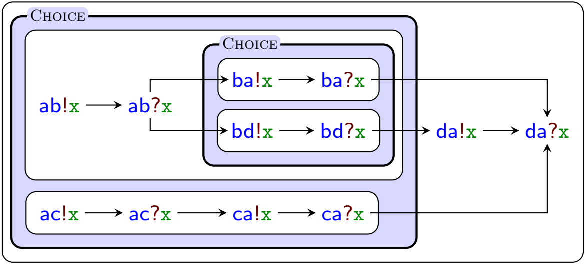
Refining: resolving (any number of) choices

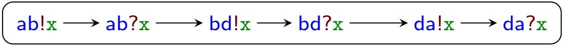




Branching pomsets

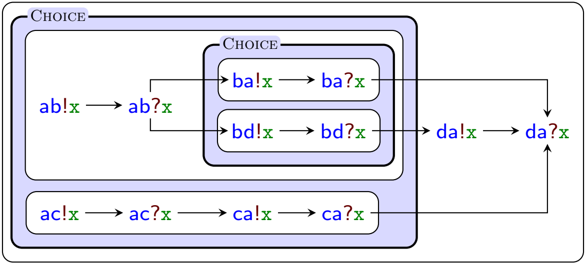
Refining: resolving (any number of) choices





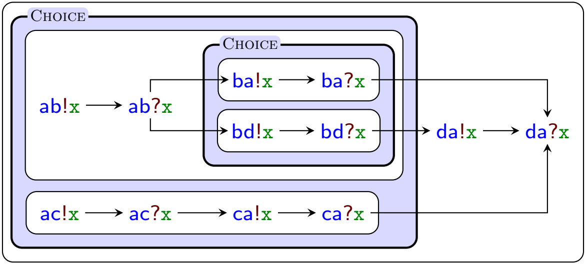
Branching pomsets

Enabling: ‘maximal’ refinement s.t. event *e* is minimal and top-level

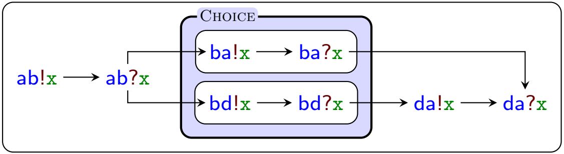


Branching pomsets

Enabling: 'maximal' refinement s.t. event *e* is minimal and top-level

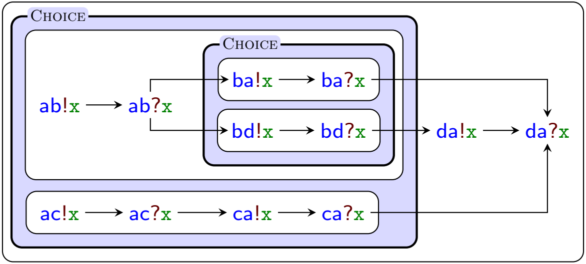


Enabling e_1 ($\lambda(e_1) = \mathsf{ab!x}$)

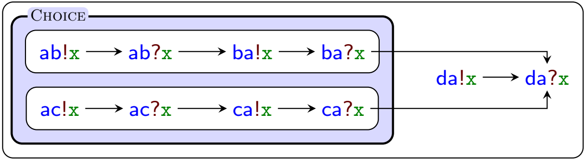


Branching pomsets

Enabling: 'maximal' refinement s.t. event e is minimal and top-level

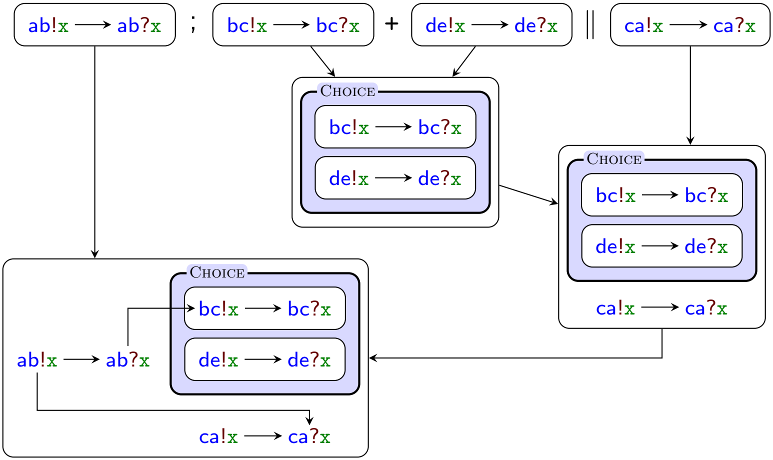


Enabling e_{11} ($\lambda(e_{11}) =$ da!x)



Branching pomsets for choreographies

$$[\![a\to b{:}x \;;\; ((b\to c{:}x + d\to e{:}x) \parallel c\to a{:}x)]\!]$$



Branching pomsets for choreographies

Theorem

If [...] then choreography c is bisimilar to branching pomset $[\![c]\!]$.

Branching pomsets for choreographies

Theorem

If [...] then choreography c *is bisimilar to branching pomset* $[\![c]\!]$.

Lemma

If [...] and $c_2 \xrightarrow{\ell} c_2'$ *and* $[\![c_2]\!] \xrightarrow{\ell} [\![c_2']\!]$ *then* $c_1 \xrightarrow{\checkmark_\ell} c_1'$ *if and only if* $[\![c_1 \,;\, c_2]\!]$ *can enable the corresponding event* e*.*

Conclusions and future work

Summary

- Branching pomsets
- Compact for both concurrency and choice
- Can express the same behaviour as choreographies

Future work

- Framework improvements: *n*-ary choices, partial order, loops
- Static analysis: realisability

`https://arca.di.uminho.pt/b-pomset/`