

# Specification of labelled reconfigurable graphs in Marge

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**José Proença**

*based on work with Alexandre Madeira, Manuel Martins, David Tinoco @ Univ. Aveiro, Portugal*

ReacTS @ SEFM, Aveiro, 5 November 2024

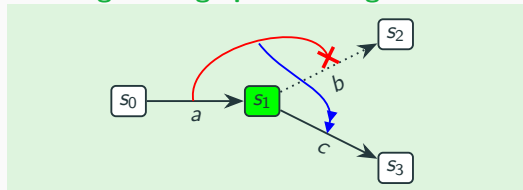
CISTER & U.Porto, Porto, Portugal



**CISTER** - Research Centre in  
Real-Time & Embedded  
Computing Systems

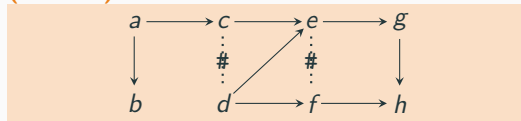


## Reconfigurable graphs in Marge



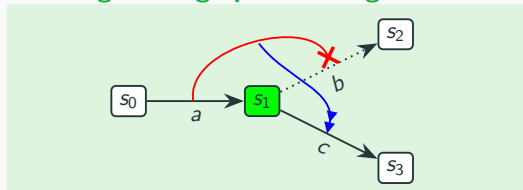
by Tinoco, Madeira, Martins, Proença  
[FACS'24]

## (Bundle) Event structures



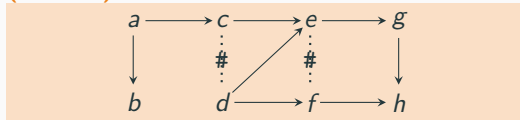
by Nielsen, Plotkin, and Winskel [TCS'81]  
and Langerak [FORTE'92]

## Reconfigurable graphs in Marge



by Tinoco, Madeira, Martins, Proença  
[FACS'24]

## (Bundle) Event structures



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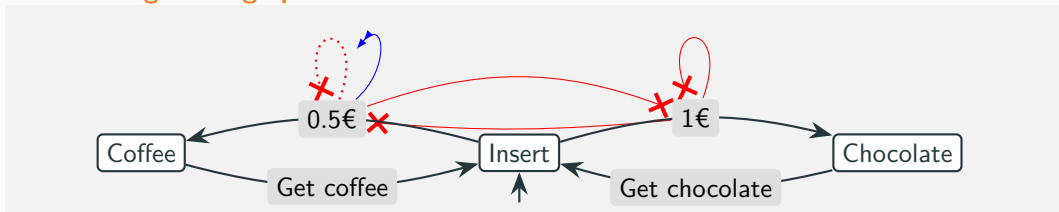
### Goal:

Get insights over reconfigurable graphs with **Marge**;  
Investigate **dependencies** & **conflicts** between reconfigurable graphs

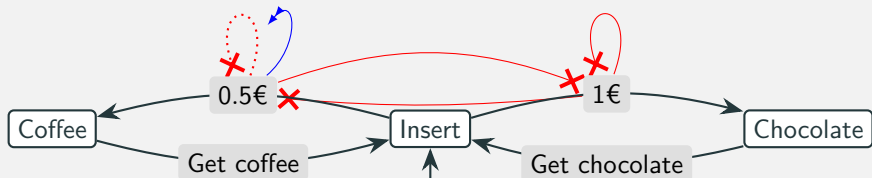
# Reconfigurable Graphs

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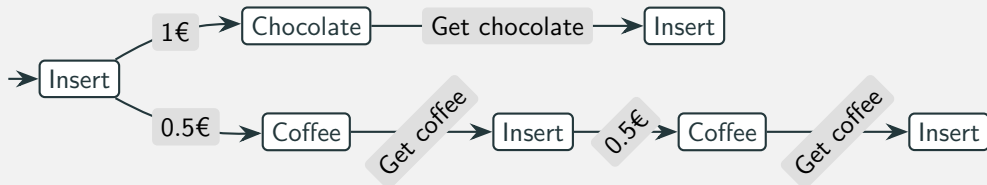
## The reconfigurable graph

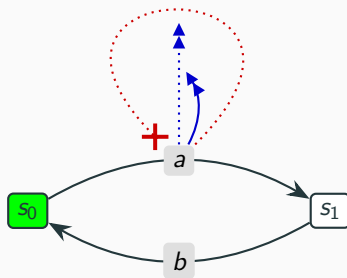


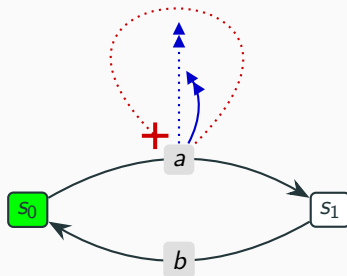
## The reconfigurable graph



## can be encoded as







When can these be useful?

Tool to experiment with semantics/compositions:

<https://fm-dcc.github.io/marge>



- Developed in Scala, using CAOS (generating JavaScript with ScalaJS)
- Static website that loads a compiled JavaScript (fully offline, no server)

- Input or load example
- Run step-by-step
- Run all steps
- Find possible problems
- Count states/edges

*Animator of Labelled Reactive Graphs*

Input Reactive Graphs

```
1 init s0
2 s0 --> s0 : act
3 act --! act : offAct disabled
4 act --> offAct : on1 disabled
5 act --> on1
```

turns off a transition after 3 times.

Examples

Simple Counter Penguin  
Vending (max 1eur) Vending (max 3prod)  
Intrusive product Conflict  
Dependencies Dynamic SPL

View State

Step-by-step

Trace:

undo

Enabled transitions:  
act

Step-by-step (simpler)

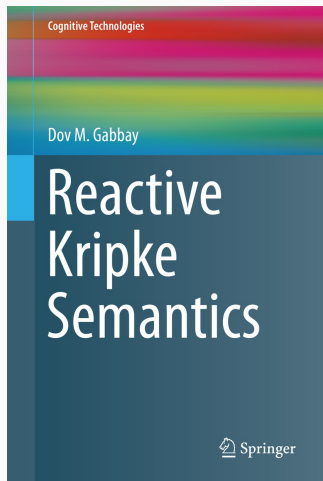
Step-by-step (txt)

All steps

Possible problems

Number of states and edges

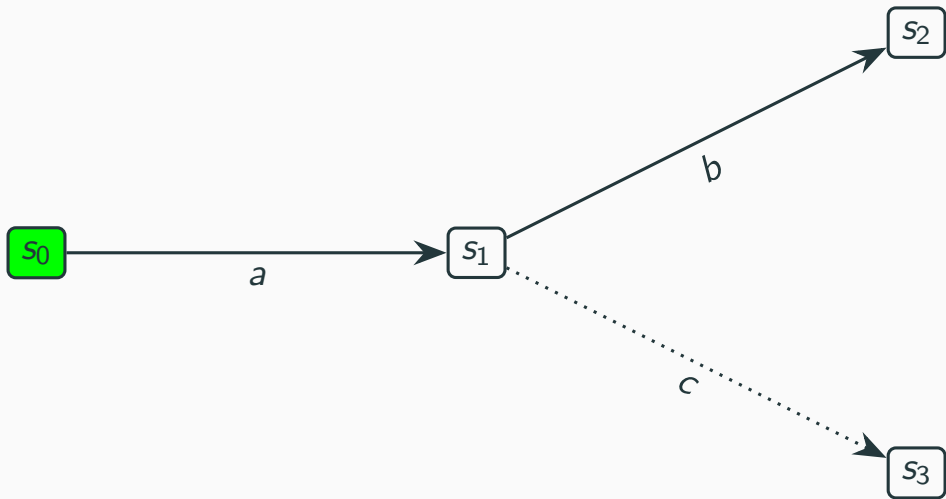
<https://fm-dcc.github.io/MARGe>

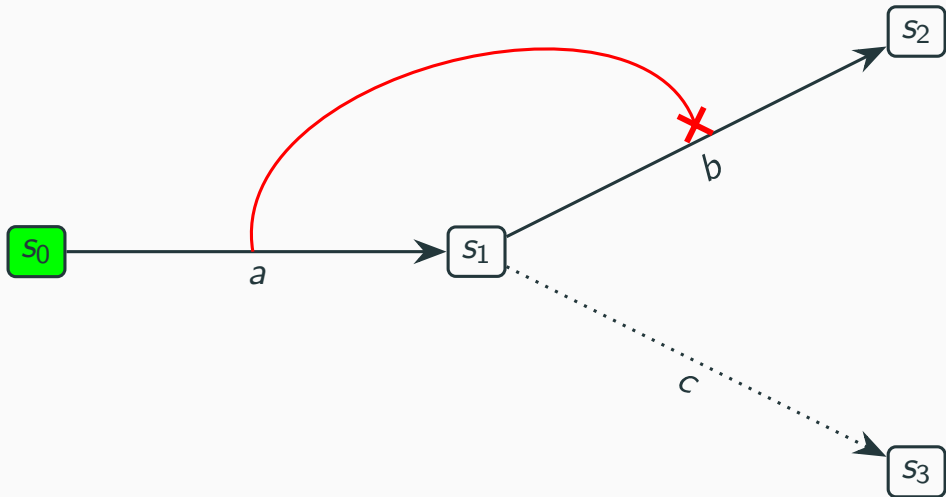


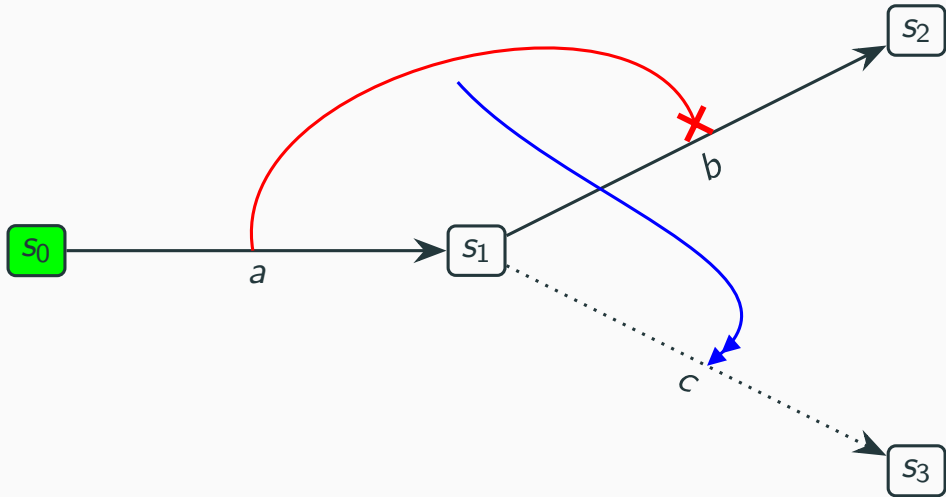
*“In computer science the word reactivity has been used to denote systems that react to their environment and are not meant to terminate, as coined by Pnueli and Harel in [On the development of reactive systems, 1985]. In this work **the word has a different meaning, reactive systems are history-dependent relational structures, where the accessibility relation is determined not only by the point where one is, but also by the previous transitions**”*

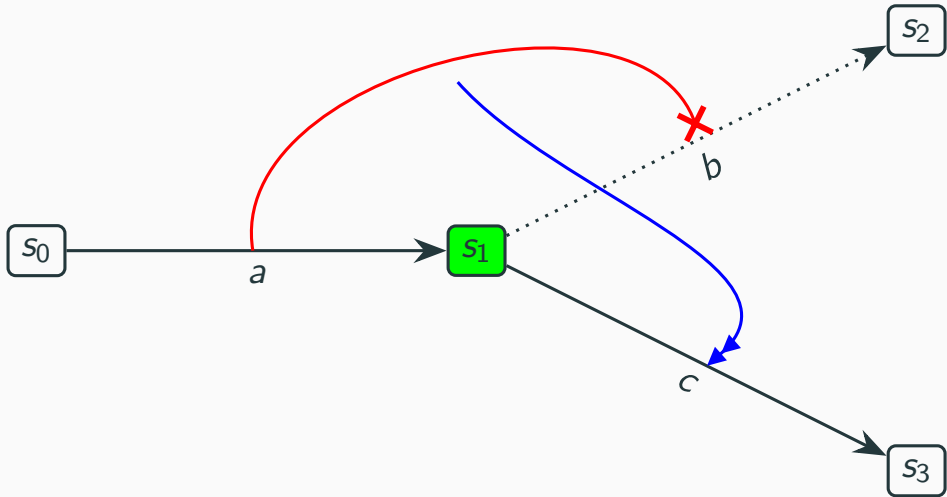
Dov M. Gabbay (2013)

I call **Reconfigurable Graph** instead of **Reactive Graph** in this talk





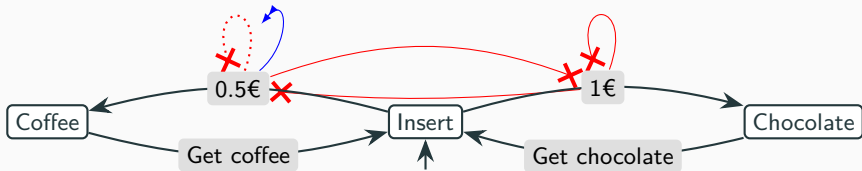




# A labelled version of a reactive graph

A **Multi-Actions Reactive Graph** is a tuple  $M = (W, Act, E, \rightarrow, \Rightarrow, \dashrightarrow, \bar{\cdot}, w_0, \alpha_0)$  where:

- $W$  – states
- $Act$  – actions
- $E$  – edges
- $w_0 \in W$  – initial state;
- $\alpha_0 \subseteq E$  – initially active edges
- $\rightarrow \subseteq W \times Act \times W$  – ground edges
- $\Rightarrow \subseteq E \times E$  – activating edges
- $\dashrightarrow \subseteq E \times E$  – deactivating edges
- $\bar{\cdot} : E \rightarrow (\rightarrow \cup \Rightarrow \cup \dashrightarrow)$  – internal details of edges



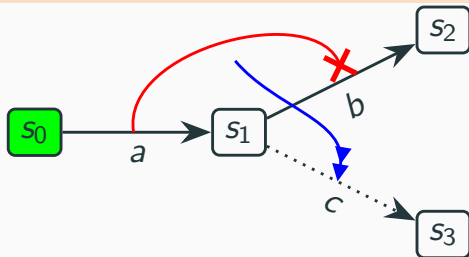
A reconfigurable graph  $M$  can evolve its configuration  $\langle w_0, \alpha_0 \rangle$  by the rule

$$\frac{\exists e \in \alpha \quad \cdot \quad \bar{e} = w \xrightarrow{a} w' \quad \wedge \quad \alpha' = (\alpha \cup \text{on}(e, \alpha)) \setminus \text{off}(e, \alpha)}{\langle w, \alpha \rangle \xrightarrow{a}_M \langle w', \alpha' \rangle}$$



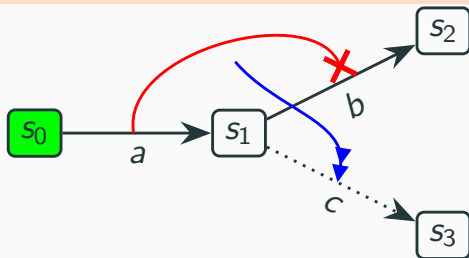
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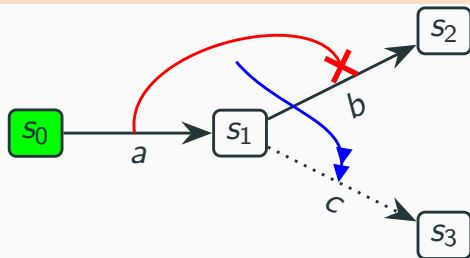
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$$\langle s_0, \{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \rangle \xrightarrow{a}$$

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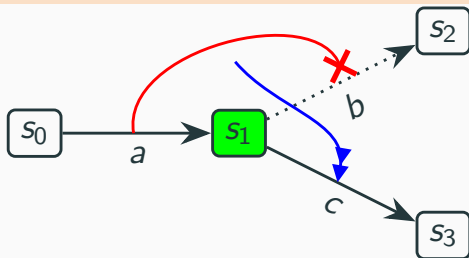
$$\frac{\exists e \in \alpha \quad \cdot \quad \bar{e} = w \xrightarrow{a} w' \quad \wedge \quad \alpha' = (\alpha \cup \text{on}(e, \alpha)) \setminus \text{off}(e, \alpha)}{\langle w, \alpha \rangle \xrightarrow{a}_M \langle w', \alpha' \rangle}$$



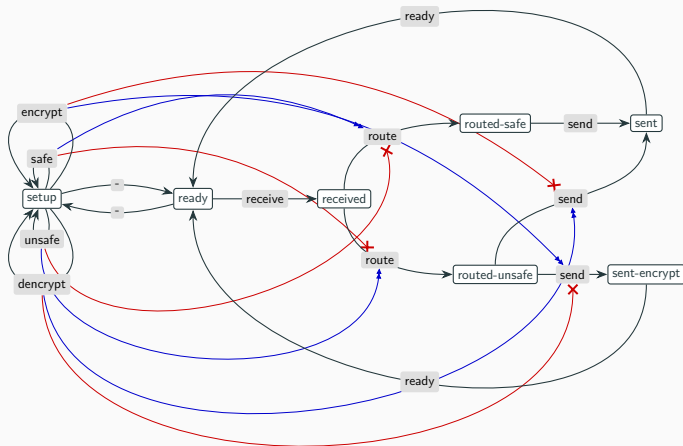
$$\langle s_0, \{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \rangle \xrightarrow{a} \langle s_1, (\{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \cup \{s_1 \xrightarrow{c} s_3\}) \setminus \{s_1 \xrightarrow{b} s_2\} \rangle$$

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$$\langle s_0, \{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \rangle \xrightarrow{a} \langle s_1, (\{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{c} s_3, \dots\}) \rangle$$



Example adapted from Cordy, Classen, Heymans, Legay, Schobbens: *Model checking adaptive software with featured transition systems* (ASAS 2013).

## Composition of models

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## Goal

Help building **complex systems** by composing simpler modules.

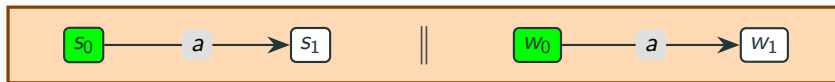
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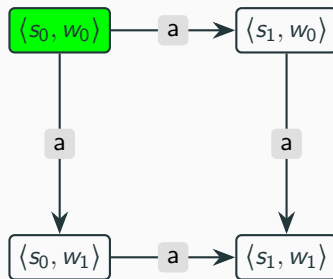
## Four products of reactive graphs

- **asynchronous** and synchronous
- **with** and **without** intrusive transitions

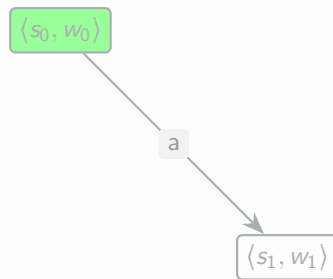


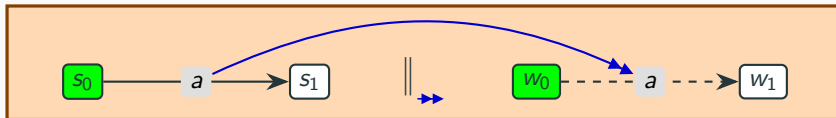


Asynchronous (interleaving)

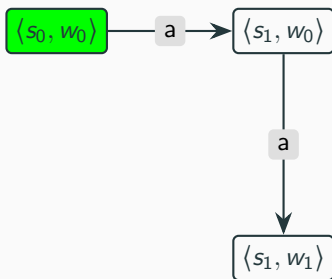


Synchronous (communicating)

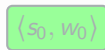


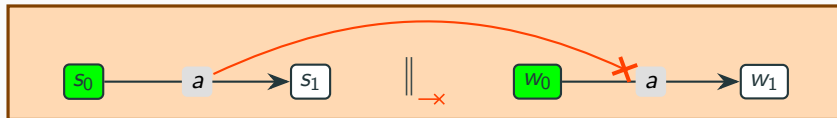


**Asynchronous** (interleaving)

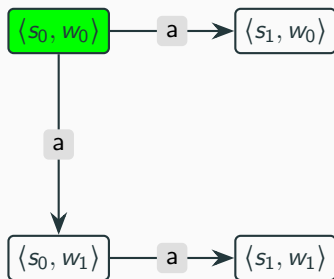


**Synchronous** (communicating)

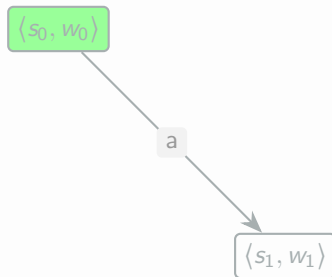


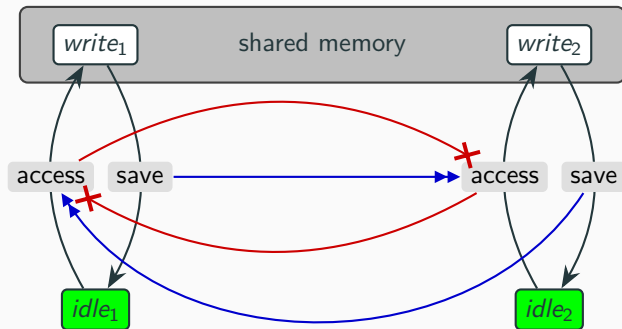


Asynchronous



Synchronous





## Asynchronous product with intrusive transitions

Given  $c_1 = \langle s_1, \alpha_1 \rangle$  and  $c_2 = \langle s_2, \alpha_2 \rangle$ , the product,  $c_1 \parallel_{\Gamma^\oplus, \Gamma^\ominus} c_2$  is defined as follows:

$$\alpha_i(\Gamma^\oplus, \Gamma^\ominus, e) = (\alpha_i \cup \text{on}(e, \alpha_i) \cup \Gamma^\oplus(e)) \setminus (\text{off}(e, \alpha_i) \cup \Gamma^\ominus(e))$$

$$\frac{\exists e \in \alpha_1 \cdot \bar{e} = s_1 \xrightarrow{a} s'_1 \wedge \alpha'_1 = (\alpha_1 \cup \text{on}(e, \alpha_1)) \setminus \text{off}(e, \alpha_1) \wedge \alpha'_2 = \alpha_2(\Gamma^\oplus, \Gamma^\ominus, e)}{\langle s_1, \alpha_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s_2, \alpha_2 \rangle \xrightarrow{a} \langle s'_1, \alpha'_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s_2, \alpha'_2 \rangle}$$

$$\frac{\exists e \in \alpha_2 \cdot \bar{e} = s_2 \xrightarrow{a} s'_2 \wedge \alpha'_2 = (\alpha_2 \cup \text{on}(e, \alpha_2)) \setminus \text{off}(e, \alpha_2) \wedge \alpha'_1 = \alpha_1(\Gamma^\oplus, \Gamma^\ominus, e)}{\langle s_1, \alpha_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s_2, \alpha_2 \rangle \xrightarrow{a} \langle s_1, \alpha'_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s'_2, \alpha'_2 \rangle}$$

A **NETWORK of Multi-Actions Reactive Graph** is a tuple  $M = (W, Act, E, \rightarrow, \blacktriangleright, \dashrightarrow, \bar{\cdot}, W_0, \alpha_0)$ :

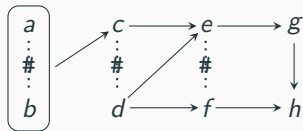
- $W$  – states
- $Act$  – actions
- $E$  – edges
- $W_0 \subseteq W$  – (multi)set of initial states;
- $\alpha_0 \subseteq E$  – initially active edges
- $\rightarrow \subseteq W \times Act \times W$  – ground edges
- $\blacktriangleright \subseteq E \times E$  – activating edges
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- $\bar{\cdot} : E \rightarrow (\rightarrow \cup \blacktriangleright \cup \dashrightarrow)$  – internal details of edges

**Evolving a configuration**  $\langle W_0, \alpha_0 \rangle$

$$\frac{\exists e \in \alpha \quad \cdot \quad \bar{e} = w \xrightarrow{a} w' \quad \wedge \quad \alpha' = (\alpha \cup \text{on}(e, \alpha)) \setminus \text{off}(e, \alpha)}{\langle W \cup \{w\}, \alpha \rangle \xrightarrow{a} \langle W \cup \{w'\}, \alpha' \rangle}$$

# Event Structures

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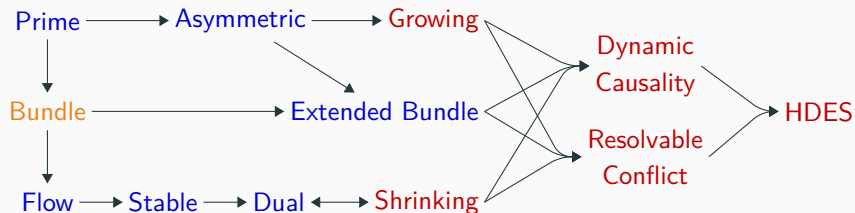
## Valid traces

- $a \cdot c \cdot e \cdot g \cdot h$
- $d \cdot b \cdot e \cdot g \cdot h$
- $d \cdot f \cdot a \cdot h$
- ...

- $a \cdots \# \cdots b$  –  $a$  excludes  $b$
- $a \xrightarrow{\text{red}} b$  –  $b$  must come after  $a$
- $\{a, b\} \xrightarrow{\text{red}} c$  –  $c$  must come after  $a$  and  $b$ , and  $a \cdots \# \cdots b$
- all events must be used



**Landscape** (partial): **static** and **dynamic** classes of event structures.



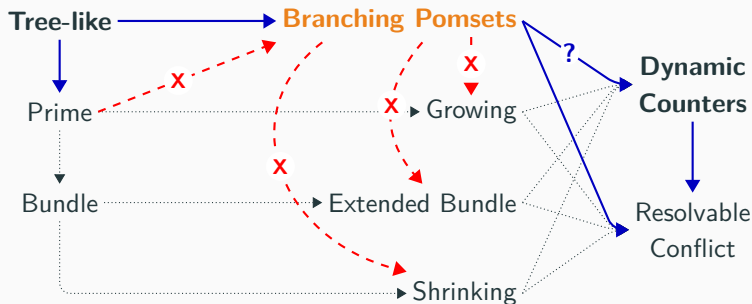
Arrows represent (strict) inclusion in terms of expressiveness

Used to represent different classes of Petri nets

*Arbach, Karcher, Peters, Nestmann, Dynamic causality in event structures*

[FORTE 2015/LMCS 2018]

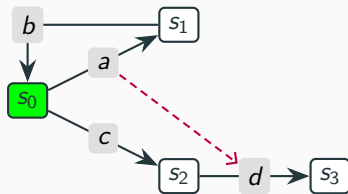
Landscape (partial): **static** and **dynamic** classes of event structures.



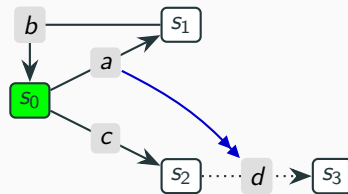
Used also to relate branching pomsets – *Edixhoven, Jongmans, Proença, Castellani, Branching pomsets: design, expressiveness and applications to choreographies [JLAMP 2024]*

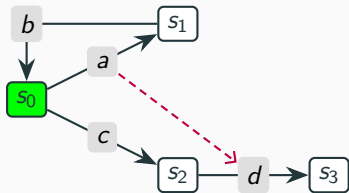
## **Dependencies as reconfigurations**

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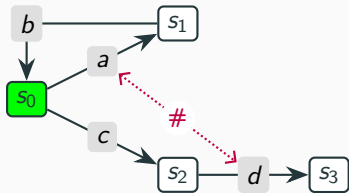
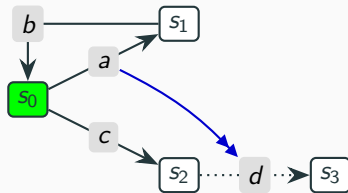


$\Rightarrow$

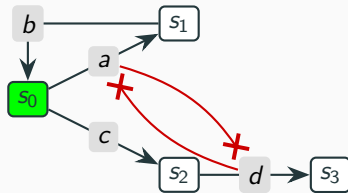


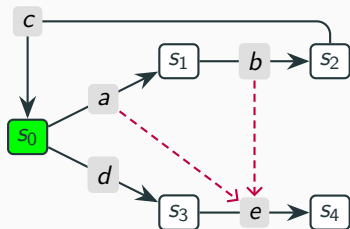


$\Rightarrow$

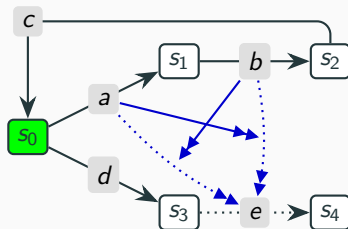


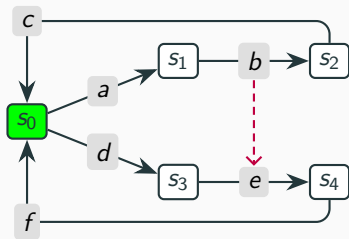
$\Rightarrow$



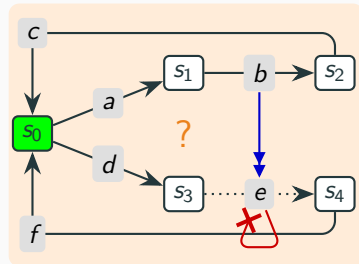


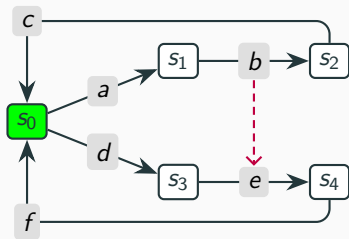
$\Rightarrow$



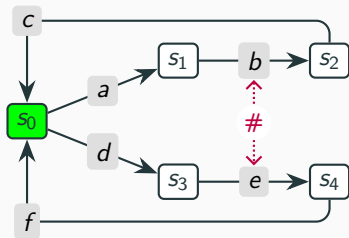
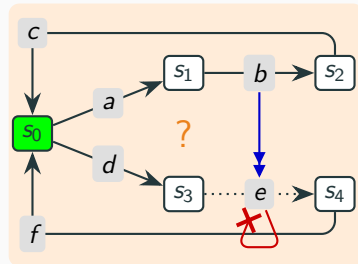


$\Rightarrow$

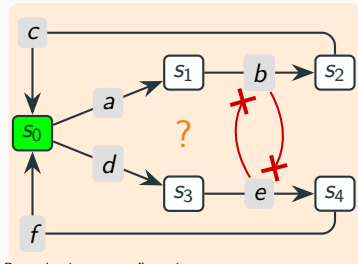




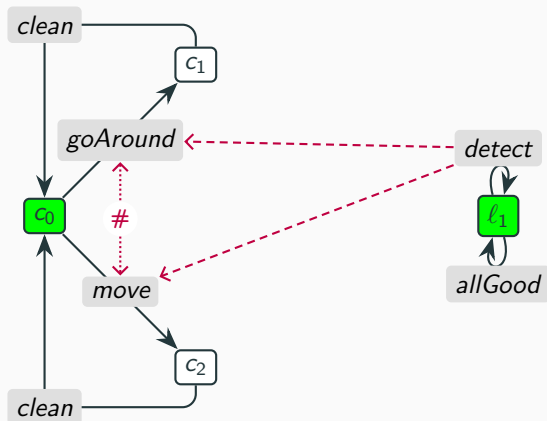
⇒

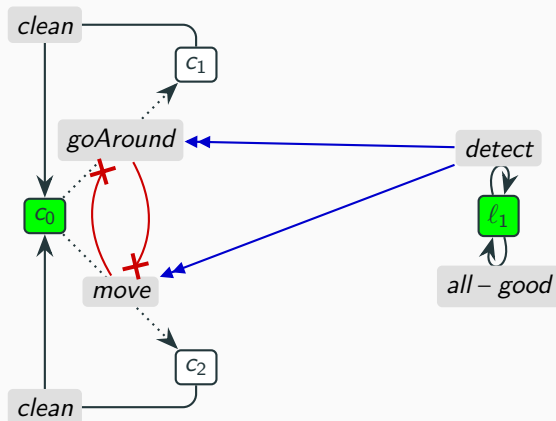


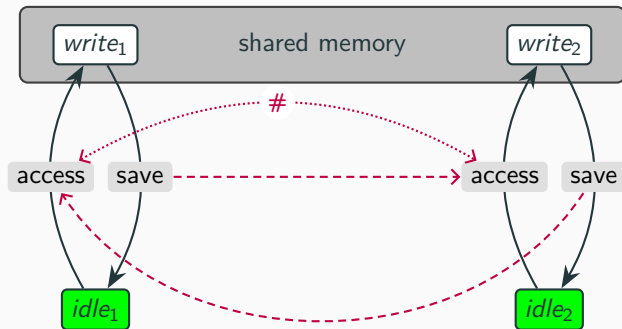
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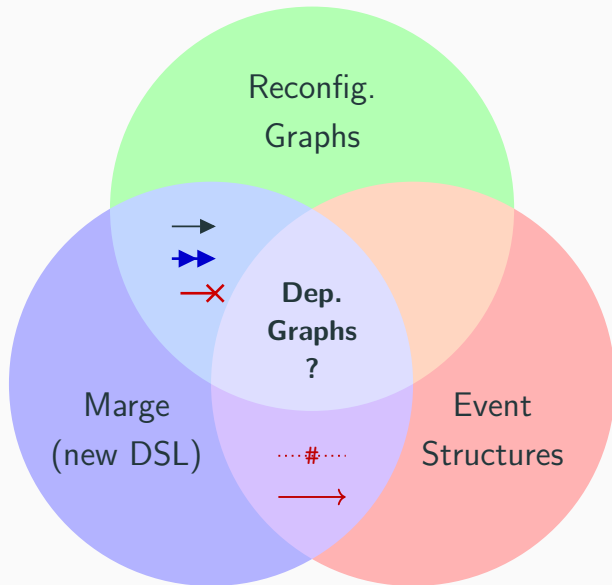




- When to **reset** a dependency/conflict?
- Different dependency notions (e.g., from different event structures)?
- Should dependability/conflict be a **primitive** in the model?  
(e.g., keep track of the number of missing activations)
- How **compositional** are these operators?
- Semantics for reconfigurable graphs (or variation) with **Petri nets**?

**Wrap up – towards dependable  
graphs in Marge**

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*“...I was told that every good presentation must have a Venn diagram” [EINAR, LIMA, ICTAC 2023]*