

Reconfigurable graphs for event structures

José Proença

based on work with Alexandre Madeira, Manuel Martins, David Tinoco @ Univ. Aveiro, Portugal

APM Workshop 2024, Turin, 2-4 October 2024

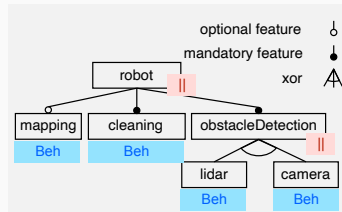
CISTER & U.Porto, Porto, Portugal



CISTER - Research Centre in
Real-Time & Embedded
Computing Systems

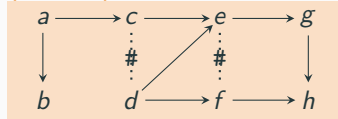


Behavioural feature models



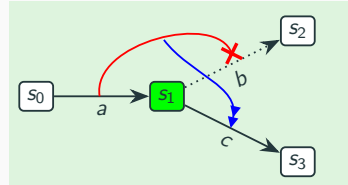
by Pässler, Fortz, ter Beek,
Damiani, Mousavi, Johnsen,
Tapia Tarifa [UNPUBLISHED]

(Bundle) Event structures



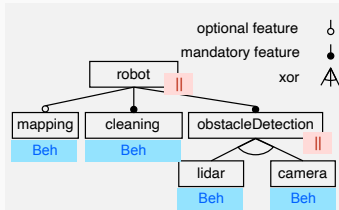
by Nielsen, Plotkin, and
Winskel [TCS'81] and
Langerak [FORTE'92]

Reconfigurable graphs



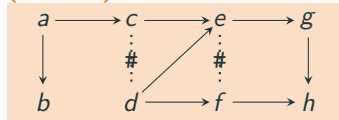
by Tinoco, Madeira, Martins,
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Behavioural feature models



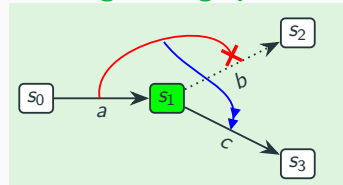
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Reconfigurable graphs

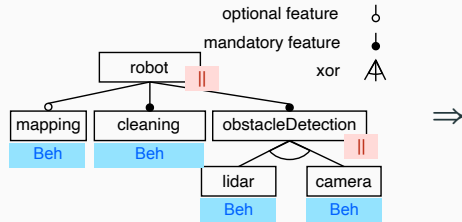


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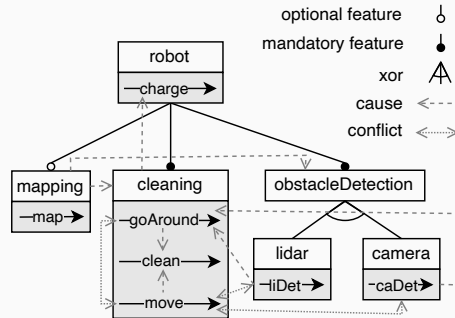
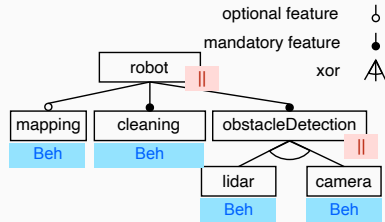
Goal:

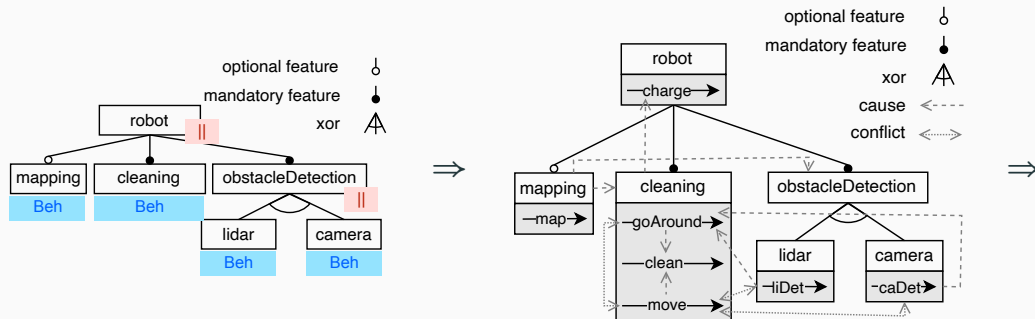
Investigate dependencies & conflicts
in (Networks of) Reconfigurable graphs

Behavioural Feature Models



Behavioural Feature Model of a Cleaning Robot



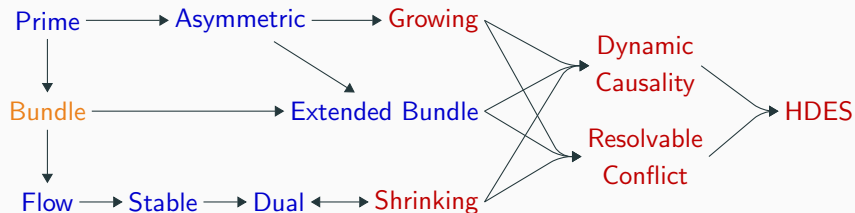


Family of Event Structures

- called *Featured Event Structure*
- Events labelled with feature conditions
- Similar to *featured transition systems* (more traditional)

Event Structures

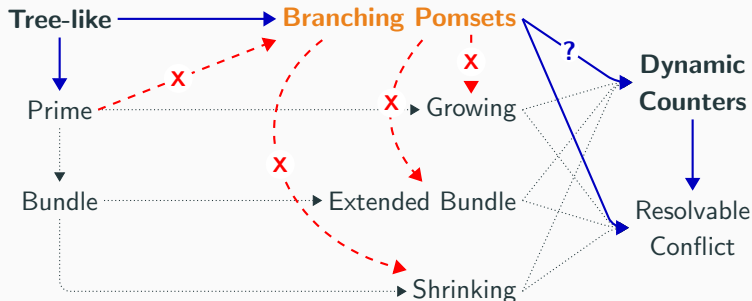
Landscape (partial): **static** and **dynamic** classes of event structures.



Arrows represent (strict) inclusion in terms of expressiveness

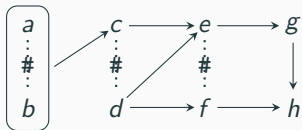
Arbach, Karcher, Peters, Nestmann, Dynamic causality in event structures
[FORTE 2015/LMCS 2018]

Landscape (partial): **static** and **dynamic** classes of event structures.



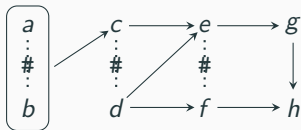
Arrows represent (strict) inclusion in terms of expressiveness

Used also to relate branching pomsets – *Edixhoven, Jongmans, Proença, Castellani, Branching pomsets: design, expressiveness and applications to choreographies [JLAMP 2024]*



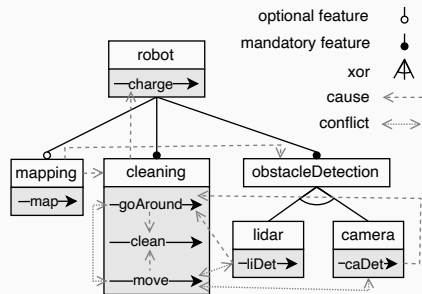
Valid traces

- $a \cdot c \cdot e \cdot g \cdot h$
- $d \cdot b \cdot e \cdot g \cdot h$
- $d \cdot f \cdot a \cdot h$
- ...



Valid traces

- $a \cdot c \cdot e \cdot g \cdot h$
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- ...

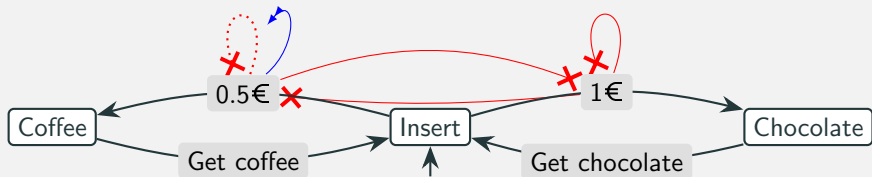


Valid traces

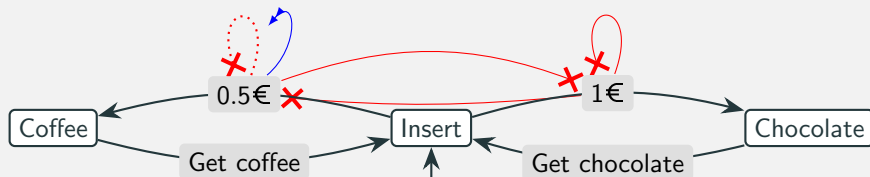
- $(\text{map})? \cdot \text{move} \cdot \text{clean} \cdot \text{charge}$
- $(\text{map})? \cdot \text{liDet} \cdot \text{goAround} \cdot \text{clean} \cdot \text{charge}$
- $(\text{map})? \cdot \text{caDet} \cdot \text{goAround} \cdot \text{clean} \cdot \text{charge}$

Reconfigurable Graphs

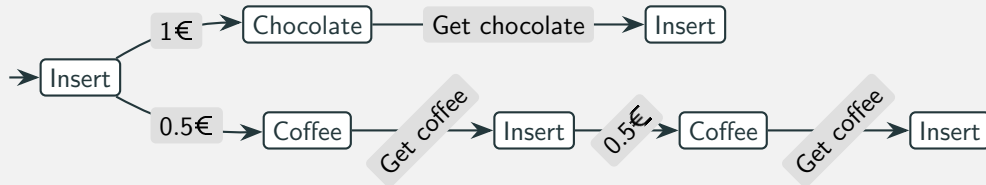
The reconfigurable graph

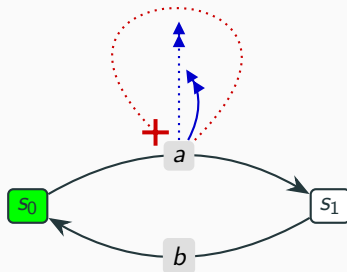


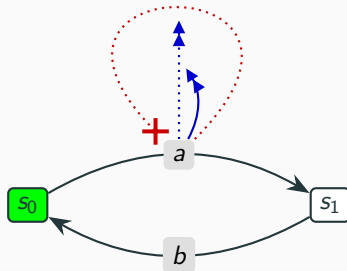
The reconfigurable graph



can be encoded as



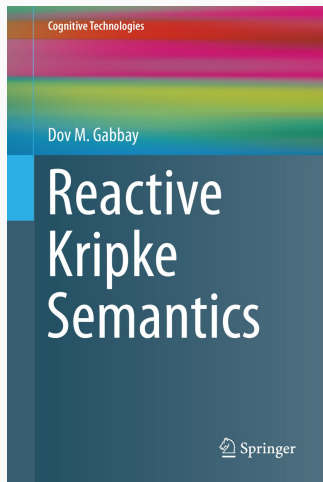




When can these be useful?

Tool to experiment with semantics/compositions:

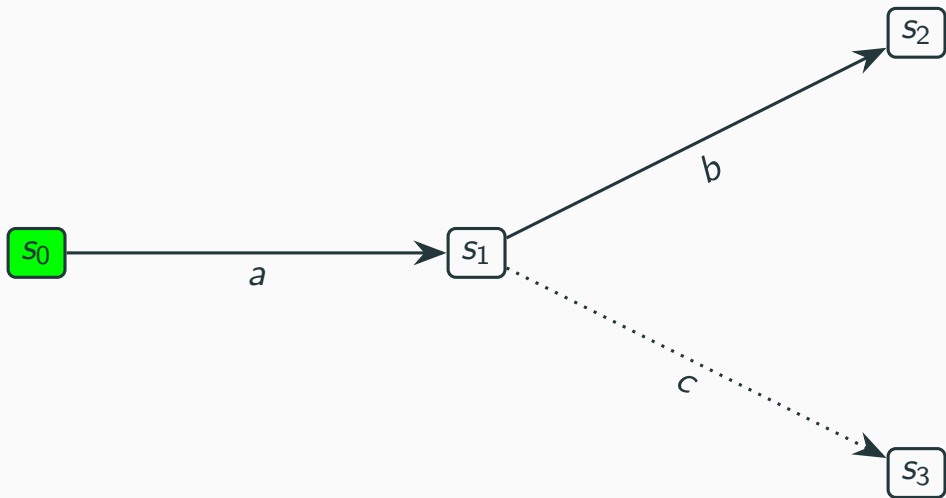
<https://fm-dcc.github.io/marge>

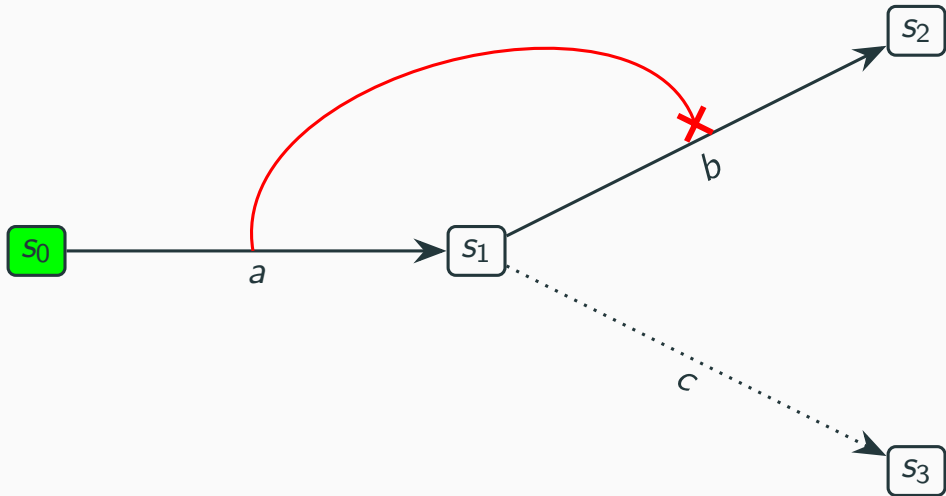


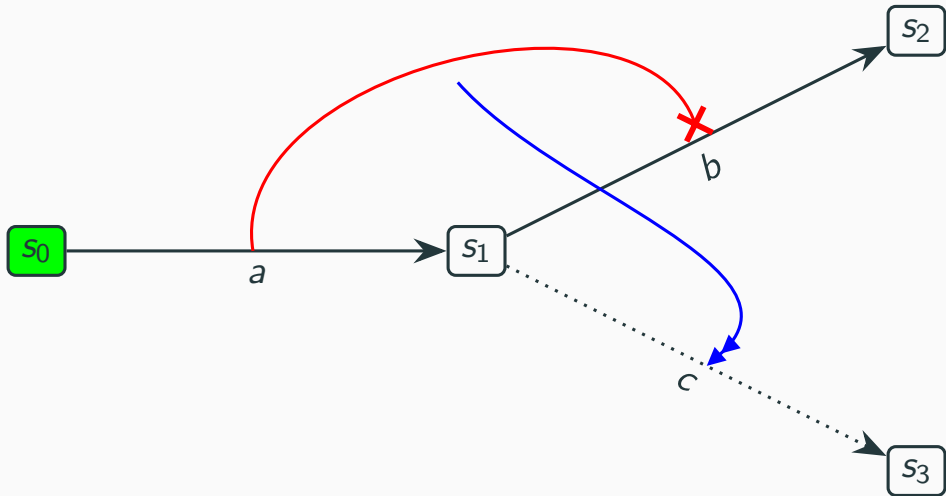
"In computer science the word reactivity has been used to denote systems that react to their environment and are not meant to terminate, as coined by Pnueli and Harel in [On the development of reactive systems, 1985]. In this work the word has a different meaning, reactive systems are history-dependent relational structures, where the accessibility relation is determined not only by the point where one is, but also by the previous transitions"

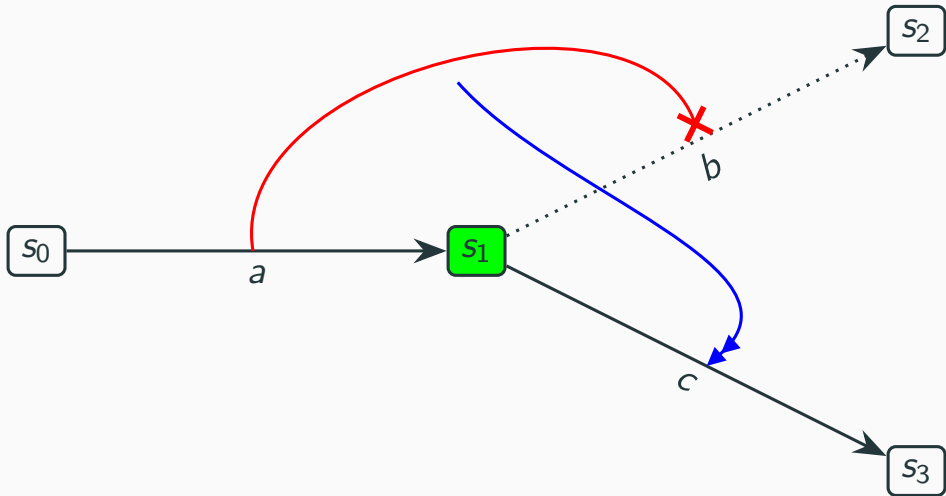
Dov M. Gabbay (2013)

I call **Reconfigurable Graph** instead of **Reactive Graph** in this talk



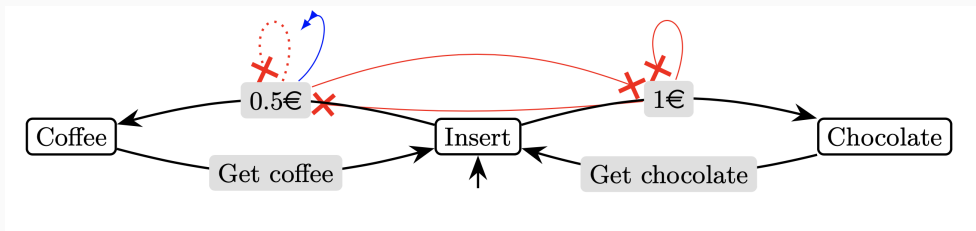






A Multi-Actions Reactive Graph is a tuple $M = (W, Act, E, \rightarrow, \Rightarrow, \rightarrow\!\times, \bar{\cdot}, w_0, \alpha_0)$ where:

- W – states
- Act – actions
- E – edges
- $w_0 \in W$ – initial state;
- $\alpha_0 \subseteq E$ – initially active edges
- $\rightarrow \subseteq W \times Act \times W$ – ground edges
- $\Rightarrow \subseteq E \times E$ – activating edges
- $\rightarrow\!\times \subseteq E \times E$ – deactivating edges
- $\bar{\cdot} : E \rightarrow (\rightarrow \cup \Rightarrow \cup \rightarrow\!\times)$ – internal details of edges

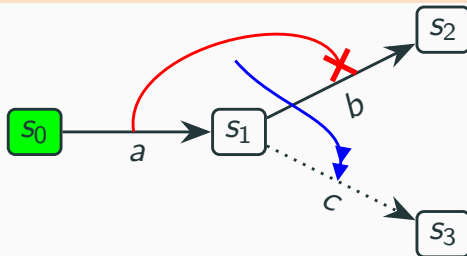


A reconfigurable graph M can evolve its configuration $\langle w_0, \alpha_0 \rangle$ by the rule

$$\frac{\exists e \in \alpha \quad \cdot \quad \bar{e} = w \xrightarrow{a} w' \quad \wedge \quad \alpha' = (\alpha \cup \text{on}(e, \alpha)) \setminus \text{off}(e, \alpha)}{\langle w, \alpha \rangle \xrightarrow{a}_M \langle w', \alpha' \rangle}$$

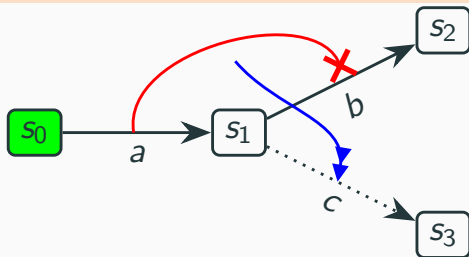
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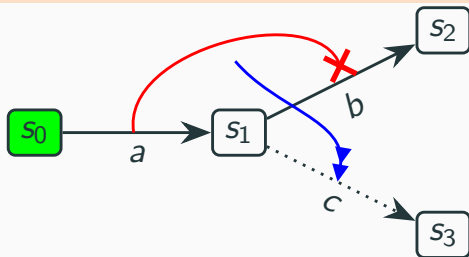
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$$\langle s_0, \{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \rangle \xrightarrow{a}$$

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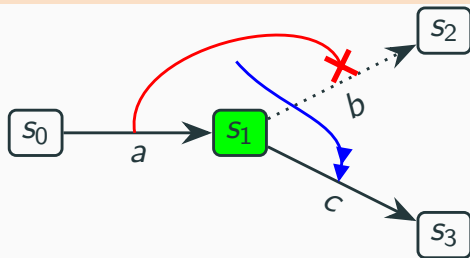
$$\frac{\exists e \in \alpha \quad \cdot \quad \bar{e} = w \xrightarrow{a} w' \quad \wedge \quad \alpha' = (\alpha \cup \text{on}(e, \alpha)) \setminus \text{off}(e, \alpha)}{\langle w, \alpha \rangle \xrightarrow{a}_M \langle w', \alpha' \rangle}$$



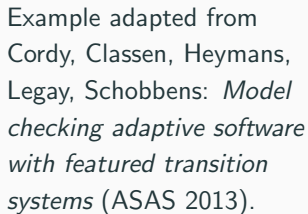
$$\langle s_0, \{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \rangle \xrightarrow{a} \langle s_1, (\{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \cup \{s_1 \xrightarrow{c} s_3\}) \setminus \{s_1 \xrightarrow{b} s_2\} \rangle$$

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$$\frac{\exists e \in \alpha \quad \cdot \quad \bar{e} = w \xrightarrow{a} w' \quad \wedge \quad \alpha' = (\alpha \cup \text{on}(e, \alpha)) \setminus \text{off}(e, \alpha)}{\langle w, \alpha \rangle \xrightarrow{a}_M \langle w', \alpha' \rangle}$$



$$\langle s_0, \{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{b} s_2, \dots\} \rangle \xrightarrow{a} \langle s_1, (\{s_0 \xrightarrow{a} s_1, s_1 \xrightarrow{c} s_3, \dots\}) \rangle$$



Composition of models

Goal

Help building **complex systems** by composing simpler modules.

Goal

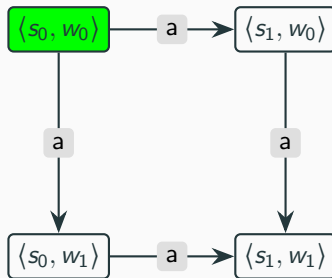
Help building **complex systems** by composing simpler modules.

Four products of reactive graphs

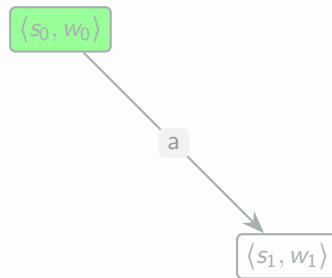
- **asynchronous** and synchronous
- **with** and **without** intrusive transitions

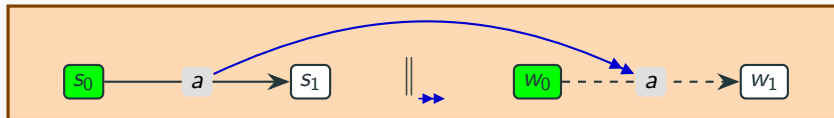


Asynchronous (interleaving)

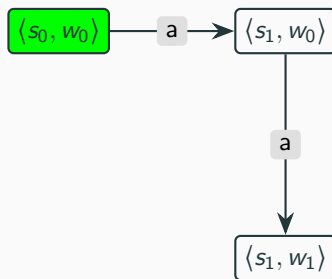


Synchronous (communicating)

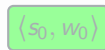


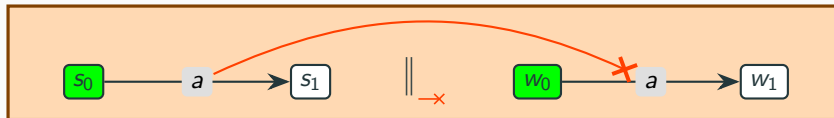


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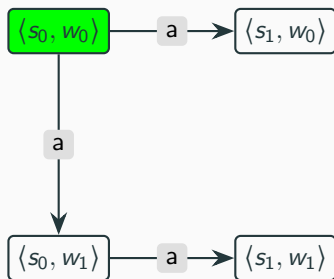


Synchronous (communicating)

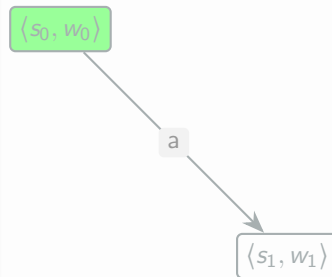


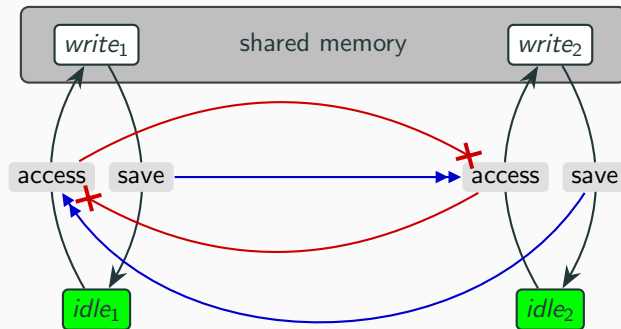


Asynchronous



Synchronous





Asynchronous product with intrusive transitions

Given $c_1 = \langle s_1, \alpha_1 \rangle$ and $c_2 = \langle s_2, \alpha_2 \rangle$, the product, $c_1 \parallel_{\Gamma^\oplus, \Gamma^\ominus} c_2$ is defined as follows:

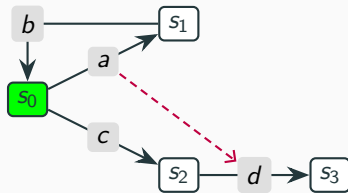
$$\alpha_i(\Gamma^\oplus, \Gamma^\ominus, e) = (\alpha_i \cup \text{on}(e, \alpha_i) \cup \Gamma^\oplus(e)) \setminus (\text{off}(e, \alpha_i) \cup \Gamma^\ominus(e))$$

$$\frac{\exists e \in \alpha_1 \quad \cdot \quad \bar{e} = s_1 \xrightarrow{a} s'_1 \quad \wedge \quad \alpha'_1 = (\alpha_1 \cup \text{on}(e, \alpha_1)) \setminus \text{off}(e, \alpha_1) \quad \wedge \quad \alpha'_2 = \alpha_2(\Gamma^\oplus, \Gamma^\ominus, e)}{\langle s_1, \alpha_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s_2, \alpha_2 \rangle \xrightarrow{a} \langle s'_1, \alpha'_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s_2, \alpha'_2 \rangle}$$

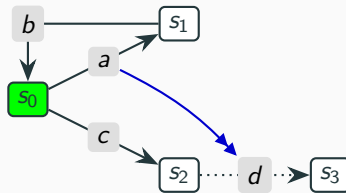
$$\frac{\exists e \in \alpha_2 \quad \cdot \quad \bar{e} = s_2 \xrightarrow{a} s'_2 \quad \wedge \quad \alpha'_2 = (\alpha_2 \cup \text{on}(e, \alpha_2)) \setminus \text{off}(e, \alpha_2) \quad \wedge \quad \alpha'_1 = \alpha_1(\Gamma^\oplus, \Gamma^\ominus, e)}{\langle s_1, \alpha_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s_2, \alpha_2 \rangle \xrightarrow{a} \langle s_1, \alpha'_1 \rangle \parallel_{\Gamma^\oplus, \Gamma^\ominus} \langle s'_2, \alpha'_2 \rangle}$$

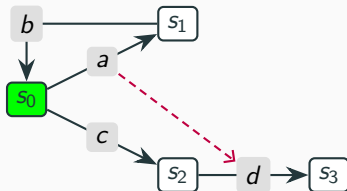
Tool support: <https://fm-dcc.github.io/MARGe>

Dependencies as reconfigurations

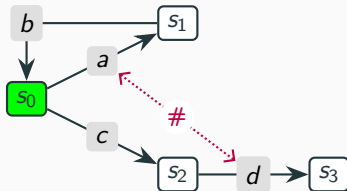
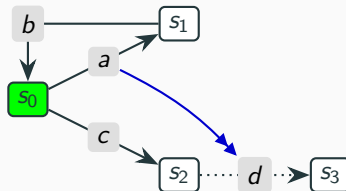


\Rightarrow

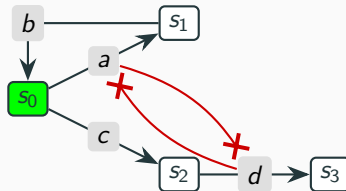


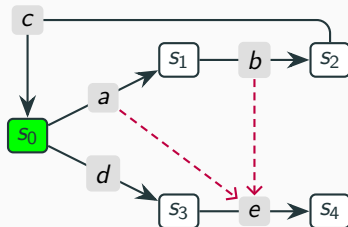


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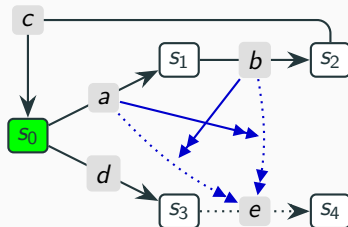


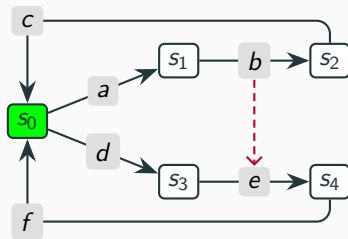
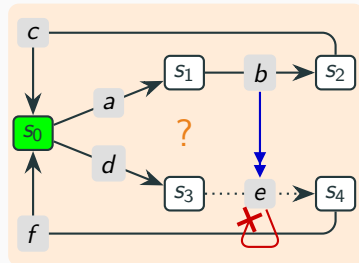
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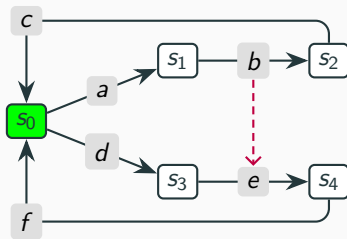




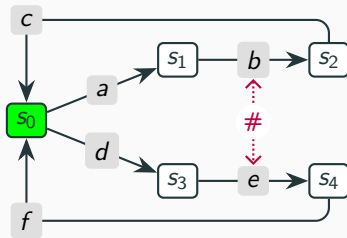
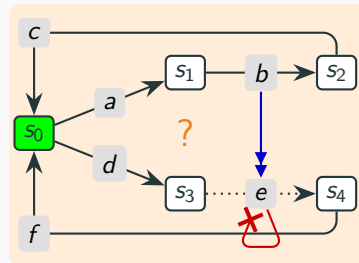
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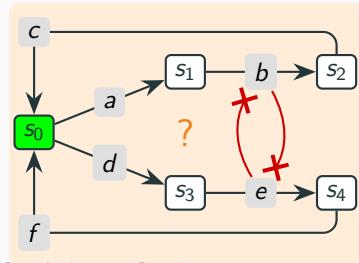
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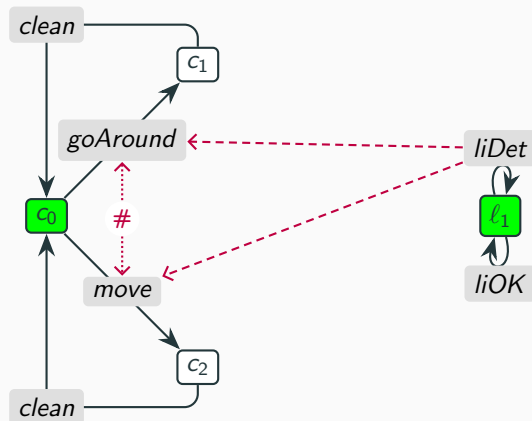


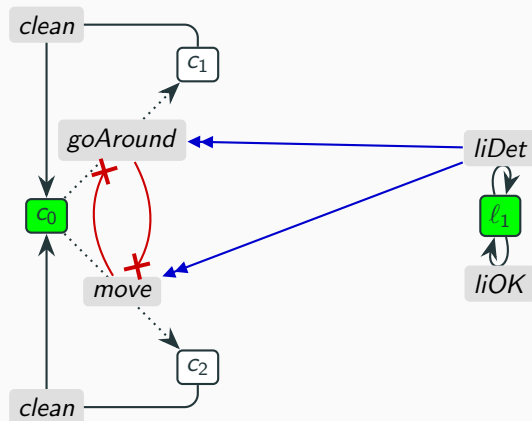
\Rightarrow

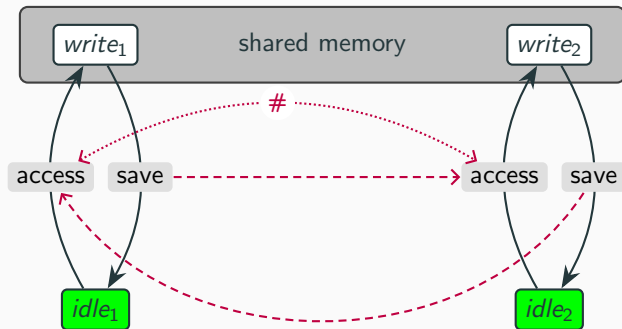


\Rightarrow





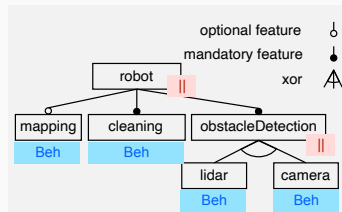




- When to **reset** a dependency/conflict?
- Different dependency notions (e.g., from different event structures)?
- Should dependability/conflict be a **primitive** in the model?
- How **compositional** are these operators?
- Semantics for reconfigurable graphs (or variation) with **Petri nets**?

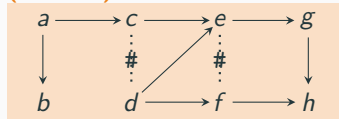
Wrap up – towards dependable graphs

Behavioural feature models



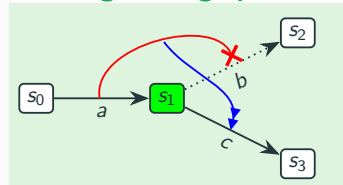
*Pässler, Fortz, ter Beek,
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(Bundle) Event structures



*by Nielsen, Plotkin, and
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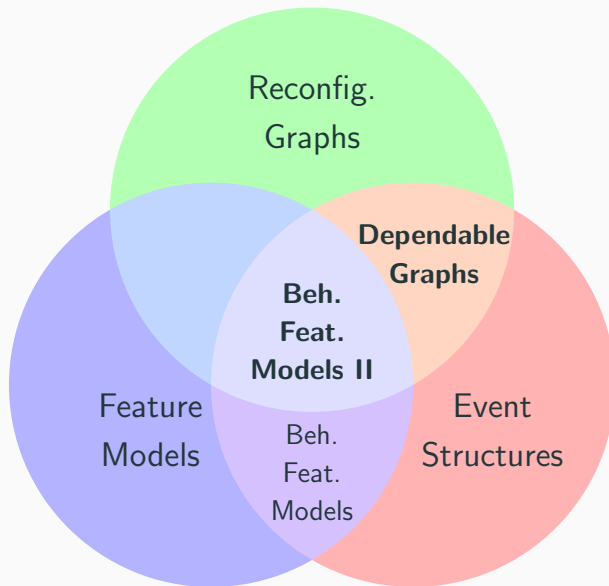
Reconfigurable graphs



*Tinoco, Madeira, Martins,
Proença [FACS'24]*

Experiments:

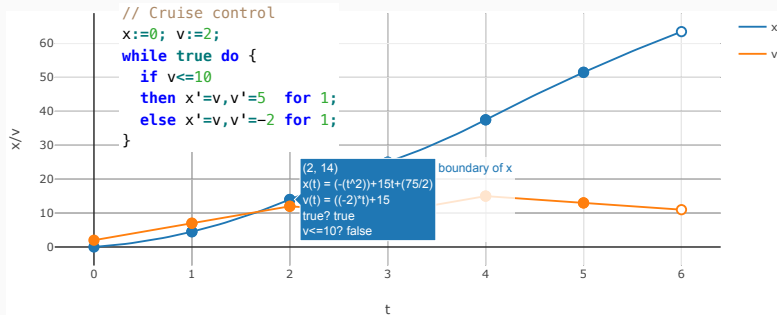
Modelling dependencies & conflicts
using (Networks of) Reconfigurable graphs



"...I was told that every good presentation must have a Venn diagram" [EINAR, LIMA, ICTAC 2023]

Lince (prototype tool – <http://arcatools.org/lince>)

- Simple while-language with differential equations
- Precise simulation using a symbolic solver (SageMath)



Improved Lince accepted at FMAS @ iFM'24

- More operators outside differential equations
- Complex examples (following a target, overtake an object, oscillation, etc.)
- Custom visualisations (2D/3D path, etc.)
- New approximated simulator